

BENCHMARK YIELD CURVES

The following table shows price spreads, as percentage, of a selected group of instruments of French and German benchmark instruments. Notice that these spreads derive from convergence of instrument yields to the yield curve. These differentials are useful to portfolio managers more than to arbitragers. Arbitrage is calculated as a reversal to its historical mean, not to a yield curve. Notice the heatmap for preferences

Price Spread differential (percent) on €100 long and short positions

PSpd	FR	FR	FR	FR	FR	FR	FR	FR	FR	FR
DE	1.54%	1.49%	1.41%	1.18%	1.16%	1.14%	1.09%	1.07%	0.89%	0.87%
FR	1.47%	1.41%	1.33%	1.11%	1.09%	1.07%	1.02%	1.00%	0.81%	0.80%
FR	1.33%	1.27%	1.19%	0.96%	0.95%	0.93%	0.87%	0.85%	0.67%	0.66%
DE	1.15%	1.09%	1.01%	0.78%	0.76%	0.75%	0.69%	0.67%	0.49%	0.47%
FR	1.05%	0.99%	0.91%	0.69%	0.67%	0.65%	0.59%	0.57%	0.39%	0.38%
FR	0.94%	0.88%	0.80%	0.58%	0.56%	0.54%	0.49%	0.47%	0.28%	0.27%
DE	0.88%	0.82%	0.74%	0.52%	0.50%	0.48%	0.43%	0.41%	0.22%	0.21%
DE	0.86%	0.81%	0.73%	0.50%	0.48%	0.46%	0.41%	0.39%	0.21%	0.19%
DE	0.85%	0.79%	0.71%	0.49%	0.47%	0.45%	0.40%	0.38%	0.19%	0.18%

A rather smooth French yield curve and a wrinkled German one constitute the two classic benchmarks of the Euro Zone. We need benchmarks to price instruments, calculate possible arbitrage opportunities, term relative value and performance, portfolio preferences and Value at Risk estimation. It looks as if we will have to use both curves simultaneously as the French curve is parsimonious but 27 bps cheaper. On the other hand the creased and bumpy German curve generates difficulties for analysis. Both curves have a relatively high degree of persistence in their shapes.

Yield Migration

Yields of French cohort of instruments shifted down 107 bps on average during the past 25 weeks, half of the shift occurred during the past 6 weeks. What we can see in the French and German curves shifted down 107 and 120 bps respectively. On average the German curve is 27bps dearer the French one but half of the difference was created during these past 25 weeks.

In the process some distortions in the curves disappeared new ones were created and others augmented their gap as it can be seen in charts and tables below. The dimple in FR6Y disappeared the one in FR7Y stagnated and the one on FR15Y exacerbated.

Contrary to that, The German group of instruments presented only on serious deviation 25 weeks ago in the 10Y and 15Y area. Now the whole curve seems to be in disarray

Yield Migration

FR	Yield	25W	6W	3W	1W	1D
FR30Y	3.67%	(82)	(46)	(23)	(6)	(4)
FR20Y	3.85%	(68)	(42)	(21)	(5)	(3)
FR15Y	3.62%	(80)	(47)	(25)	(8)	(5)
FR10Y	3.36%	(81)	(47)	(26)	(8)	(4)
FR9Y	3.27%	(83)	(48)	(26)	(8)	(4)
FR8Y	3.14%	(88)	(49)	(27)	(10)	(5)
FR7Y	2.95%	(98)	(54)	(30)	(12)	(7)
FR6Y	2.84%	(102)	(55)	(30)	(12)	(7)
FR5Y	2.68%	(113)	(60)	(32)	(14)	(6)
FR4Y	2.48%	(124)	(63)	(33)	(13)	(5)
FR3Y	2.21%	(137)	(66)	(33)	(12)	(4)
FR2Y	1.87%	(164)	(80)	(42)	(16)	(9)
FR1Y	1.68%	(176)	(76)	(32)	(12)	(7)
		(107)	(56)	(29)	(10)	(5)

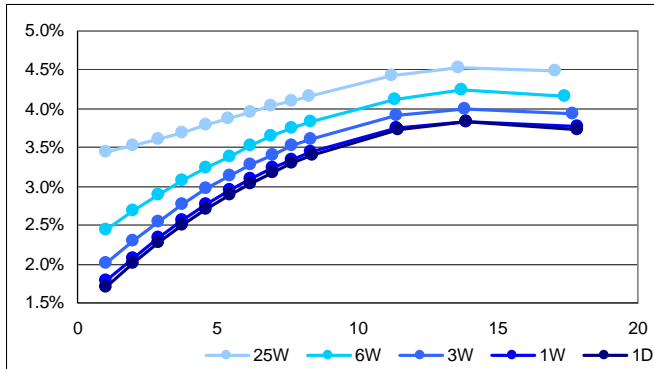
Yield Migration

DE	Yield	25W	6W	3W	1W	1D
DE30Y	3.48%	(89)	(49)	(24)	(6)	(5)
DE20Y	3.68%	(78)	(45)	(23)	(7)	(5)
DE15Y	3.48%	(90)	(48)	(25)	(7)	(5)
DE10Y	2.90%	(102)	(55)	(27)	(7)	(4)
DE9Y	2.87%	(103)	(55)	(28)	(8)	(4)
DE8Y	2.85%	(103)	(55)	(27)	(8)	(4)
DE7Y	2.71%	(109)	(56)	(28)	(9)	(5)
DE6Y	2.52%	(118)	(59)	(29)	(10)	(5)
DE5Y	2.27%	(132)	(65)	(33)	(12)	(6)
DE4Y	2.19%	(138)	(67)	(34)	(12)	(5)
DE3Y	1.81%	(160)	(76)	(37)	(12)	(5)
DE2Y	1.71%	(164)	(75)	(35)	(11)	(3)
DE1Y	1.68%	(173)	(71)	(28)	(10)	(4)
		(120)	(60)	(29)	(9)	(5)

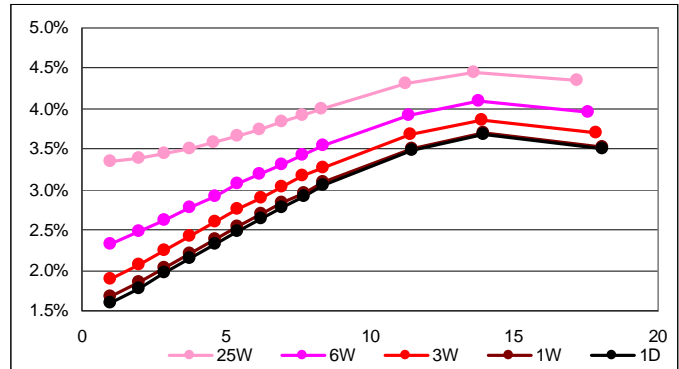
Daily Market Report

5 January 2009

FR



DE



Fair Yield Migration

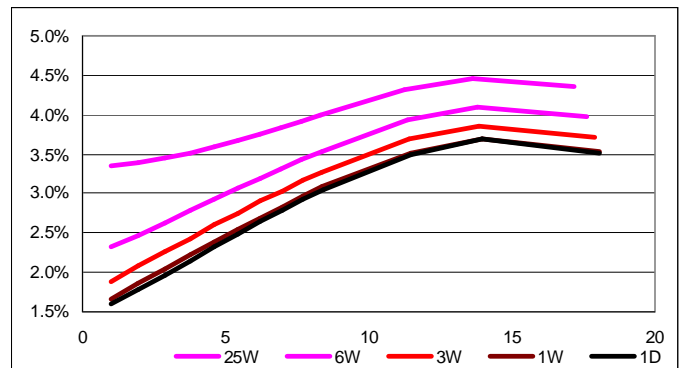
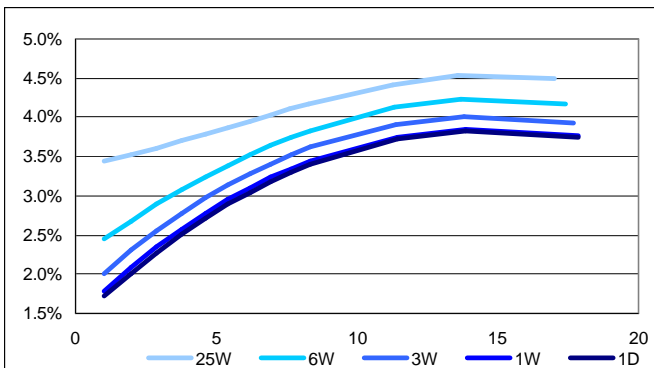
The fair yield of benchmark curves are calculated in a way that a non-arbitrage system is guaranteed. Indeed it shifts and tilts as the original curve does. Half of the shift in curves occurred during the past 6 weeks. On average the German curve is 27bps dearer than the French one but half of the difference was created during these past 25 weeks.

Fair Yield Migration

FR	FYield	25W	6W	3W	1W	1D
FR30Y	3.70%	(79)	(46)	(23)	(6)	(3)
FR20Y	3.79%	(74)	(44)	(21)	(5)	(4)
FR15Y	3.68%	(74)	(44)	(23)	(6)	(4)
FR10Y	3.35%	(82)	(48)	(26)	(9)	(5)
FR9Y	3.25%	(85)	(49)	(27)	(10)	(5)
FR8Y	3.13%	(90)	(51)	(28)	(10)	(5)
FR7Y	2.99%	(96)	(53)	(29)	(11)	(5)
FR6Y	2.83%	(104)	(56)	(30)	(12)	(6)
FR5Y	2.65%	(113)	(59)	(32)	(12)	(6)
FR4Y	2.45%	(125)	(63)	(33)	(13)	(6)
FR3Y	2.21%	(140)	(68)	(34)	(13)	(6)
FR2Y	1.94%	(158)	(74)	(35)	(13)	(7)
FR1Y	1.64%	(179)	(80)	(37)	(13)	(7)

Fair Yield Migration

DE	FYield	25W	6W	3W	1W	1D
DE30Y	3.46%	(89)	(51)	(26)	(8)	(5)
DE20Y	3.64%	(82)	(45)	(22)	(6)	(5)
DE15Y	3.45%	(86)	(48)	(24)	(7)	(5)
DE10Y	3.00%	(100)	(54)	(28)	(8)	(5)
DE9Y	2.88%	(104)	(56)	(29)	(9)	(5)
DE8Y	2.74%	(109)	(58)	(30)	(9)	(5)
DE7Y	2.60%	(115)	(60)	(30)	(10)	(5)
DE6Y	2.44%	(122)	(62)	(31)	(10)	(5)
DE5Y	2.28%	(131)	(65)	(32)	(11)	(4)
DE4Y	2.11%	(140)	(67)	(32)	(11)	(4)
DE3Y	1.93%	(151)	(70)	(33)	(11)	(4)
DE2Y	1.74%	(165)	(73)	(33)	(11)	(4)
DE1Y	1.56%	(180)	(76)	(33)	(11)	(4)



Yield Pricing cost sheet

The tables show the yield cost sheet for each instrument. Cost prices are derived from analysing the yield curve and the properties of constituent instruments, the properties being the derivatives. The first and second derivatives have Christian names, Duration and Convexity, whereas the third has no name. We just call it 'the third'.

Cost components are a fixed annual percentage, the cost of duration, the usual rebate originated by convexity and the cost of the third derivative of price to yield function. In a normal situation all components would have a positive sign except for convexity that would be negative.

Please observe that the French curve looks normal whereas the German one has inverted signs. Sign inversion is associated with high volatility. Difference with normal volatility calculation is that the present technique needs only a sample of one day

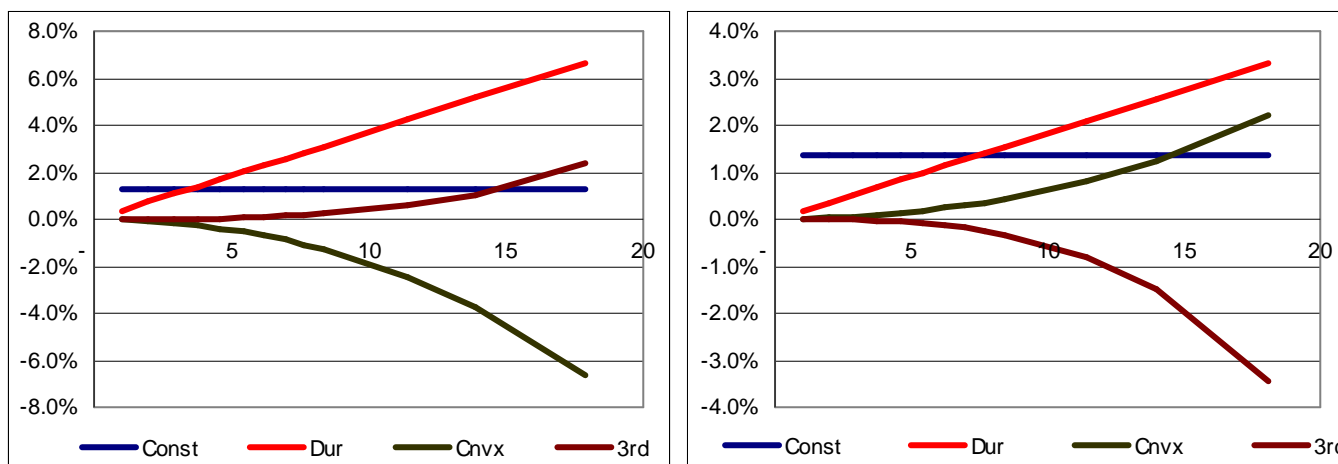
Yield Pricing cost sheet (FR chain)

FR	Duration	Convexity	Third	1.3E-02	3.7E-03	-1.6E-04	2.9E-06	Yld	Est	Pricing	pct
FR30Y	17.92	419	8,161	1.30%	6.64%	-6.61%	2.37%	3.70%	3.67%	(3)	-0.52%
FR20Y	13.90	235	3,629	1.30%	5.15%	-3.71%	1.05%	3.79%	3.85%	6	0.81%
FR15Y	11.43	154	1,967	1.30%	4.24%	-2.43%	0.57%	3.68%	3.62%	(6)	-0.66%
FR10Y	8.36	81	771	1.30%	3.10%	-1.27%	0.22%	3.35%	3.36%	1	0.08%
FR9Y	7.67	68	600	1.30%	2.84%	-1.07%	0.17%	3.25%	3.27%	2	0.15%
FR8Y	6.95	56	453	1.30%	2.58%	-0.88%	0.13%	3.13%	3.14%	1	0.07%
FR7Y	6.20	45	329	1.30%	2.30%	-0.71%	0.10%	2.99%	2.95%	(4)	-0.24%
FR6Y	5.42	35	227	1.30%	2.01%	-0.55%	0.07%	2.83%	2.84%	1	0.05%
FR5Y	4.61	25	147	1.30%	1.71%	-0.40%	0.04%	2.65%	2.68%	3	0.13%
FR4Y	3.76	18	86	1.30%	1.39%	-0.28%	0.03%	2.45%	2.48%	3	0.11%
FR3Y	2.88	11	44	1.30%	1.07%	-0.17%	0.01%	2.21%	2.21%	-	0.00%
FR2Y	1.96	6	18	1.30%	0.73%	-0.09%	0.01%	1.94%	1.87%	(7)	-0.13%
FR1Y	1.00	2	5	1.30%	0.37%	-0.03%	0.00%	1.64%	1.68%	4	0.04%

Yield Pricing cost sheet (DE chain)

DE	Duration	Convexity	Third	1.4E-02	1.8E-03	5.2E-05	-4.1E-06	Yld	Est	Pricing	pct
DE30Y	18.13	427	8,387	1.37%	3.30%	2.22%	-3.44%	3.46%	3.48%	2	0.35%
DE20Y	13.97	238	3,684	1.37%	2.55%	1.24%	-1.51%	3.64%	3.68%	3	0.41%
DE15Y	11.46	155	1,983	1.37%	2.09%	0.81%	-0.81%	3.45%	3.48%	4	0.44%
DE10Y	8.40	82	786	1.37%	1.53%	0.43%	-0.32%	3.00%	2.90%	(9)	-0.73%
DE9Y	7.70	69	610	1.37%	1.40%	0.36%	-0.25%	2.88%	2.87%	(1)	-0.07%
DE8Y	6.97	56	458	1.37%	1.27%	0.29%	-0.19%	2.74%	2.85%	11	0.75%
DE7Y	6.21	45	332	1.37%	1.13%	0.23%	-0.14%	2.60%	2.71%	11	0.67%
DE6Y	5.43	35	229	1.37%	0.99%	0.18%	-0.09%	2.44%	2.52%	8	0.42%
DE5Y	4.61	26	148	1.37%	0.84%	0.13%	-0.06%	2.28%	2.27%	(1)	-0.05%
DE4Y	3.76	18	87	1.37%	0.69%	0.09%	-0.04%	2.11%	2.19%	9	0.33%
DE3Y	2.88	11	45	1.37%	0.52%	0.06%	-0.02%	1.93%	1.81%	(12)	-0.34%
DE2Y	1.96	6	18	1.37%	0.36%	0.03%	-0.01%	1.74%	1.71%	(3)	-0.06%
DE1Y	1.00	2	5	1.37%	0.18%	0.01%	0.00%	1.56%	1.68%	12	0.12%

Notice how convexity and 'the third' behave differently in the French and German curves



Component pricing coefficients

Here we show how yield volatility components evolved over time. Please note how the German convexity reduced its coefficient by an order of magnitude larger with respect to the 3 week average.

Component Pricing

FR	Today	25W	6W	3W	1W	1D
ONRate	1.30E-02	-2.0E-02	-8.8E-03	-3.8E-03	-1.3E-03	-7.5E-04
Duration	3.71E-03	3.0E-03	9.3E-04	1.2E-04	-5.8E-05	4.1E-05
Convexity	-1.58E-04	-2.1E-04	-6.2E-05	5.9E-06	1.8E-05	-6.4E-07
Tercera	2.90E-06	6.0E-06	1.7E-06	-4.0E-07	-7.2E-07	-9.5E-09

Component Pricing

DE	Today	25W	6W	3W	1W	1D
ONRate	1.37E-02	-2.0E-02	-8.0E-03	-3.3E-03	-1.1E-03	-4.3E-04
Duration	1.82E-03	2.0E-03	3.5E-04	-8.1E-05	-2.8E-05	-2.2E-06
Convexity	5.20E-05	-1.1E-04	-3.2E-06	2.2E-05	1.1E-05	-1.9E-07
Tercera	-4.10E-06	2.9E-06	-2.1E-07	-8.6E-07	-4.3E-07	6.6E-09

Yield volatility

This calculation of volatility only confirms what we already saw in the yield curve coefficient analysis. French curve stays calm and smooth. The Germans are in disarray.

Yield Volatility

FR	25W	6W	3W	1W	1D
FR30Y	-0.09%	-0.11%	-0.10%	-0.08%	-0.03%
FR20Y	0.00%	-0.01%	-0.01%	0.01%	0.06%
FR15Y	-0.11%	-0.12%	-0.12%	-0.11%	-0.06%
FR10Y	-0.05%	-0.05%	-0.05%	-0.04%	0.01%
FR9Y	-0.04%	-0.04%	-0.04%	-0.03%	0.02%
FR8Y	-0.05%	-0.05%	-0.05%	-0.04%	0.01%
FR7Y	-0.10%	-0.11%	-0.11%	-0.10%	-0.04%
FR6Y	-0.06%	-0.06%	-0.06%	-0.05%	0.01%
FR5Y	-0.04%	-0.04%	-0.04%	-0.03%	0.03%
FR4Y	-0.04%	-0.04%	-0.04%	-0.03%	0.03%
FR3Y	-0.07%	-0.08%	-0.07%	-0.06%	0.00%
FR2Y	-0.14%	-0.15%	-0.14%	-0.14%	-0.07%
FR1Y	-0.02%	-0.02%	-0.01%	0.00%	0.04%

Yield Volatility

DE	25W	6W	3W	1W	1D
DE30Y	-0.04%	-0.05%	-0.05%	-0.03%	0.02%
DE20Y	-0.03%	-0.04%	-0.04%	-0.02%	0.03%
DE15Y	-0.01%	-0.02%	-0.02%	-0.01%	0.04%
DE10Y	-0.15%	-0.15%	-0.15%	-0.14%	-0.09%
DE9Y	-0.07%	-0.07%	-0.07%	-0.06%	-0.01%
DE8Y	0.05%	0.05%	0.05%	0.06%	0.11%
DE7Y	0.05%	0.04%	0.05%	0.06%	0.11%
DE6Y	0.02%	0.01%	0.02%	0.03%	0.08%
DE5Y	-0.08%	-0.08%	-0.08%	-0.07%	-0.01%
DE4Y	0.02%	0.02%	0.02%	0.03%	0.09%
DE3Y	-0.19%	-0.20%	-0.19%	-0.18%	-0.12%
DE2Y	-0.11%	-0.11%	-0.10%	-0.09%	-0.03%
DE1Y	0.06%	0.05%	0.07%	0.08%	0.12%

Fair Yield Volatility

FR	25W	6W	3W	1W	1D
FR30Y	-0.10%	-0.11%	-0.11%	-0.09%	-0.03%
FR20Y	0.01%	-0.01%	0.00%	0.01%	0.06%
FR15Y	-0.11%	-0.12%	-0.12%	-0.10%	-0.06%
FR10Y	-0.05%	-0.06%	-0.05%	-0.04%	0.01%
FR9Y	-0.04%	-0.05%	-0.04%	-0.03%	0.02%
FR8Y	-0.05%	-0.06%	-0.06%	-0.04%	0.01%
FR7Y	-0.10%	-0.11%	-0.11%	-0.09%	-0.04%
FR6Y	-0.06%	-0.06%	-0.06%	-0.04%	0.01%
FR5Y	-0.04%	-0.04%	-0.04%	-0.03%	0.03%
FR4Y	-0.04%	-0.04%	-0.04%	-0.03%	0.03%
FR3Y	-0.06%	-0.07%	-0.07%	-0.05%	0.00%
FR2Y	-0.13%	-0.14%	-0.13%	-0.12%	-0.07%
FR1Y	-0.02%	-0.03%	-0.02%	-0.01%	0.04%

Fair Yield Volatility

DE	25W	6W	3W	1W	1D
DE30Y	-0.05%	-0.06%	-0.06%	-0.04%	0.02%
DE20Y	-0.02%	-0.03%	-0.03%	-0.02%	0.03%
DE15Y	-0.01%	-0.02%	-0.02%	-0.01%	0.04%
DE10Y	-0.15%	-0.16%	-0.15%	-0.14%	-0.09%
DE9Y	-0.07%	-0.08%	-0.07%	-0.06%	-0.01%
DE8Y	0.05%	0.04%	0.05%	0.06%	0.11%
DE7Y	0.05%	0.04%	0.04%	0.06%	0.11%
DE6Y	0.01%	0.01%	0.01%	0.03%	0.08%
DE5Y	-0.08%	-0.08%	-0.08%	-0.06%	-0.01%
DE4Y	0.02%	0.02%	0.02%	0.04%	0.09%
DE3Y	-0.19%	-0.19%	-0.18%	-0.17%	-0.12%
DE2Y	-0.10%	-0.10%	-0.09%	-0.08%	-0.03%
DE1Y	0.05%	0.05%	0.06%	0.07%	0.12%

Building a Yield Curve and its zero coupons

In neither case direct bootstrapping is possible as curves present long term arbitrage opportunities; this is more a hunting ground for portfolio managers than arbitrageurs as the latter would not have sufficient time to wait. But 2% or 3% price difference is considerable as it can be viewed in the last table.

The most popular method and the one used by the ECB is the Nelson-Siegel method to which Diebold-Li method of forecasting can be applied.

However this is not the method of our choice. For one, NS method does not guarantee a non-arbitrage term structure. But the main concern stands in the fact that the lambda coefficient is not calculated from a property of bonds but it rather is treated as a property of the curve as such. Lambda is difficult to explain other than to say that is a decay factor. But decay of what? It is decay factor of other coefficients. The fact that in the end it generates zero coupons that can calculate well the present worth of bonds is not much of an argument as a good set B-Spline can do it equally well.

Our method of choice is built from the very properties of bonds being therefore of fundamental nature. It goes along the lines of a Taylor series. The final function returns a non-arbitrage structure of yields of a collection of bonds of the same class. To that chain, bootstrapping can be applied as it can also be applied the DL method of forecasting.

The idea is not different from the process of manufacturing balanced feed. A bond is taken to be a sack of coupons with properties of yield, duration and convexity as maize has proteins, fats and fibre. The question is what it is paid for, in terms of bps of the yield, duration and convexity in the market. Once we have that, pricing an out of sample instrument is just question of looking at its duration and convexity and multiplying those by the obtained prices.

Four coefficients are obtained, a constant, the price of duration, the price of convexity and the price of the 'third'.

We do that in a loop with as much iteration as there are bonds in the sample. In the first one we use the whole lot. In successive loops we take one out of the sample by turns. In each loop we regress yields against duration and convexity. Finally we calculate averages and deviations. In the tables, arbitrage free yields, are listed for each bond under column Est. (for estimation). The arbitrage free system is guaranteed by definition as there is only one price per year-duration and only one price per convexity-unit.

There is no need for a third derivative to be included at these levels of yields. The third derivative becomes important somewhere in the long bonds (30Y) at yield higher than 14% at which point duration begins to stall and retrace forming an upward flock in the curve. This phenomenon is found in deep discount bonds in some Emerging Markets.

The fixed cost has to be understood with several factors. Opportunity cost, bond bid-offer spread, FX bid offer spreads and the opportunity cost of having positions in FX forwards